Comparison of Mamdani and Sugeno Fuzzy Inference Systems

Yogesh Piolet T

Abstract—Since many years, the question of comparison between Mamdani fuzzy inference system and Sugeno inference system has always baffled the minds of several researchers. A good number of researches have been done independently on their comparison with respect to a few specific applications. One of the major motivations behind this research is to ascertain which approach is better in general. In this paper, a detailed survey of the comparison of the Mamdani and Sugeno methods of fuzzy inference has been made. It has lead to the conclusion that Sugeno fuzzy inference system is better when compared to Mamdani fuzzy inference system. But, the major problem lies ahead. A lot of applications require Mamdani method because of its expressive capability. Hence, it would be really advantageous if a procedure to transform a Mamdani system to Sugeno system is brought out. It is also important to note that Sugeno system work only for multiple input single output systems and not for multiple input multiple output systems. This creates a problem again. To deal with the above mentioned problems, some existing solution has been surveyed. The main objective has been to transform a Mamdani fuzzy inference system into a Sugeno fuzzy inference system. It would be really interesting if the Sugeno inference system could also handle multiple input multiple output systems (if at all possible).

Index Terms— Fuzzy Inference Systems, Mamdani FIS, Sugeno FIS

• *Yogesh Piolet T is with the School of Information Technology, Indian Institute of Technology Kharagpur, Kharagpur, India-721302.*
E-mail: yogeshpiolet@yahoo.co.in

1 Introduction

Fuzzy Inference systems (FIS) are composed of three major components. It consists of a rule base, a database and a reasoning mechanism. The rule base is made up of a selection of fuzzy rules. The database defines the membership functions. The reasoning mechanism is a way of inferring a reasonable output or conclusion.

1.1 Terminology

- Gyroscope - A gyroscope is a device used for measuring the orientation of space vehicles and satellite components and maintaining them.
- Haptic devices - Haptic devices allow users to touch, feel and manipulate three-dimensional objects in virtual environments and tele-operated systems.

1.2 Literature survey

Over the past few years, researchers have been constantly focussing on the comparison of the Mamdani FIS [5] and TSK (Takagi,Sugeno and Kang) [6] FIS with respect to certain applications. The results of these researches helps us to identify the best approach.

(Jassbi et. al.,2006) applied both the Mamdani FIS and TSK FIS to the gyroscope health monitoring tool [1].This monitoring tool generated alarms indicating the different levels of criticality and indicated how severe the alarm itself was. The tool included three major components of which only the generic fault detection module was considered for their comparison.

The results of their comparison are as follows: Mamdani FIS required fourteen times more processing time than the TSK FIS. To be specific, the average time taken by the Mamdani FIS to produce one result was 4.6×10^{-4} seconds while for the TSK FIS, it was 3.2×10^{-5} seconds. The TSK FIS was found to be robust in the sense that the noise present in the input data did not seem to have any effect on the functioning of the system. More importantly, TSK FIS also produced totally different results as the input data became too much different from the original noise free data. Thus, TSK FIS reacted more strongly. (Jassbi et. al.,2006) also noted that the state between “Tolerable” and “Not tolerable” was sensed clearly by the TSK FIS by decreasing the alarm level. On the other hand, Mamdani FIS reacted completely numb and did not respond to these transitions. Thus, TSK FIS has smoother transitions than Mamdani FIS.

(Jassbi et. al.,2006) concluded that TSK FIS works better in terms of processing time, more robust and better sensitivity.

(Meitzler and Sohn, 2005) had a slightly different observation. They compared the Mamdani and Sugeno methods for the modeling of visual perception of laboratory data. The main purpose of their experiment was to determine the effectiveness of a camouflage treatment in reducing the probability of detection in the visual part of the electromagnetic spectrum at various ranges, aspect angles and lighting conditions [2].

(Meitzler and Sohn, 2005) observed that the correlation of Mamdani FIS and Sugeno FIS to the experimental data was 0.85 and 0.84 respectively. Thus, they concluded that Mamdani FIS model permitted easy variation and adjustment of the parameters.

However, it must also be observed that Sugeno FIS had correlation of 0.84 and 0.65 when two and three membership function was used, respectively. Thus, the correlation of Sugeno FIS is almost close to that of Mamdani FIS.

(Hamam and Goerganas, 2008) added more credit to the TSK FIS through their experiments on Hapto-Audio-Visual applications. The main aim of their research was to determine whether users of haptic devices really had a unique and enriching experience or was it just because they could touch and manipulate virtual objects using a new technology. They proposed taxonomy to evaluate the Quality of Experience (QOE) of Hapto-Audio-Visual applications based on a few parameters as shown below in Fig. 1.

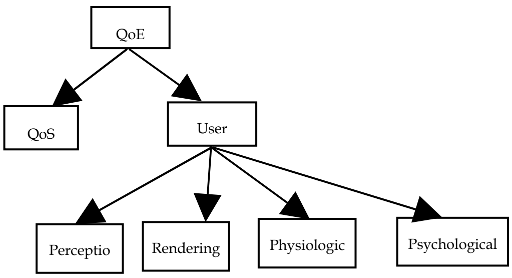

Figure 1. Organization of the QoE model (Hamam and Goerganas, 2008)

The purpose of the above QoE model is to categorize the parameters .The evaluators are then free to choose the parameters that they would like to evaluate from one or more of these categories. The perception measures informed how the user perceives the application. The rendering quality indicated the quality of graphics, audio and haptics. The physiological measures were purely the biological parameters measured directly from the user while using the applications. the psychological measures reflected the status of the user through observation.

(Hamam and Goerganas, 2008) used five parameters in the model. The parameters and their membership function are Media Synchronization (Gaussian), fatigue (Triangular), haptic rendering (Trapezoidal), degree of immersion (Triangular) and user satisfaction (Gaussian).

(Hamam and Goerganas, 2008) made the following observations. TSK FIS results were more accurate since the results that were generated were closer to what was expected. TSK FIS was found to be more dynamic to input changes. With respect to the boundary cases, TSK FIS was far more accurate than Mamdani FIS. They also observed that TSK FIS was faster than Mamdani FIS. On the other hand, Mamdani FIS displayed consistency in results and showed expressive power. However, the authors concluded that they would mostly choose TSK FIS over Mamdani FIS.

The paper is organized as follows. In section 2, the problem that is identified based on the literature survey done is explained. In section 3, the existing solution for the problem identified is described. In section 4, a suggestion is offered as to how the existing solution can be extended to the problem under consideration. In section 5, a plan for implementation is specified.

2. Problem Definition

Based on the survey done, it can be observed that Mamdani FIS is intuitive and its rule base can be easily interpreted. However, in TSK FIS, the consequents of the rules are not fuzzy. Hence, the power of interpretation is lost. But, consequents of the TSK rules can have the same number of parameters in their consequent per rule as the number of input values. This offers more flexibility to the TSK FIS over Mamdani FIS. On the other hand, Mamdani FIS can be used directly for both Multiple Input Single Output (MISO) systems and Multiple Input Multiple Output (MIMO) systems. But, TSK systems can be used only in MISO systems.

Also, note that Mamdani FIS has output membership function and an output distribution whereas TSK FIS has no output membership function nor does it have any output distribution. Rather,TSK FIS has only a resulting action. Mamdani FIS obtains a crisp result by defuzzification. On the other hand, TSK FIS has no defuzzification procedure. In TSK FIS, crisp result is obtained by using weighted average on rule's consequent.

In general, TSK FIS is always more efficient than Mamdani FIS. But, a serious drawback of the TSK FIS is that it cannot be used with MIMO systems. It is also important to note that several applications have already been developed in Mamdani FIS because of its expressive power.

A mechanism to convert the Mamdani FIS to Sugeno FIS would really help those applications already developed in Mamdani. Also, a method is to be found out to enable the TSK FIS to be used with MIMO systems.

3. Existing Solution

A solution given by (Jassbi et. al., 2007) is explained here.Their approach used the following strategy. They formalized the transformation of Mamdani FIS to TSK FIS as an optimization problem and then used three algorithms to solve this optimization problem. The three algorithms are Least Squares (LS) algorithm, Genetic Algorithm (GA) and adaptive neuro-fuzzy inference system (ANFIS) [7].

3.1 Concept of universal approximators

A universal approximation theorem can be used to identify the conditions under which a certain structure can be constructed such that it produces results which are arbitrarily close to the results of the given function. It is only an existence theorem, indicating that it only specifies the conditions under which such a structure exists. But, it is not a constructive theorem in the sense that it does provide adequate support to determine how to build the mentioned approximator.

On the basis of the above definition, it can be observed that Mamdani FIS and TSK FIS are both universal approximators.

3.2 Mapping transformation to an optimization problem

(Jassbi et. al., 2007) started with an aim to design an equivalent TSK FIS for a particular Mamdani FIS using optimization algorithm. Thus, the equivalence between Mamdani FIS and Sugeno FIS can be described as –

Mamdani output = TSK output ⇔

$$\Leftrightarrow \quad Mamdani\ output = \frac{\sum_{i=1}^{n} \alpha^i y^i}{\sum_{i=1}^{n} \alpha^i}$$

where, y^1, y^2, y^3 … y^n represent the consequents of each rule in the rule base.

α^1, α^2, α^3, … α^n represent the firing level of each rule.

4. New suggestion

It must be observed that a MIMO system can be broken down into several MISO systems which are working in parallel. Based on this idea, if we are successful in coming out with a mechanism to decompose a MIMO into a series of MISO's, then the TSK method can be directly applied on these MISO's directly. However, the mechanism to decompose a MIMO into a series of MISO's should be efficient enough and should not add additional overhead to the entire process of coming out with a solution.

It is also possible to refine the above solution to a certain extent, by carefully analyzing the operations of the heuristic algorithms to chose a best method and apply this method to convert the already existing Mamdani MISO to TSK MISO. However, to transform a Mamdani MIMO to TSK FIS, it would be necessary to convert the Mamdani MIMO to its corresponding MISO's.

5. Implementation details

A detailed analysis is expected to be carried out to determine which of the heuristic algorithm is best suited for the problem described above. Accordingly, the chosen algorithm will be applied to convert a Mamdani FIS to Sugeno FIS.

Once this phase of the implementation is complete, existing mechanisms to convert a MIMO system into a series of MISO's will be examined (if any available). Subsequently, it will be applied to the conversion of Mamdani MIMO to a series of MISO's.

5.1 Notations & problem formulation

A typical fuzzy rule in a Sugeno fuzzy model is of the follwing simplified form –

"if x is A and y is B, then z = f(x,y)"

where,
A and B are fuzzy sets in the antecedent,
z=f(x,y) is a crisp function in the consequent
f(x,y) is a polynomial of the form z=px+qy+r

For example,
Let the rule base be –
"If x is A_1 and y is B_1, then $Z_1=P_1x+Q_1y+R_1$"
"If x is A_2 and y is B_2, then $Z_2=P_2x+Q_2y+R_2$"
Suppose W_1 and W_2 are the firing levels of each of these rules separately, then the overall output is obtained via the weighted average –

$$Z = \frac{W_1Z_1 + W_2Z_2}{W_1 + W_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(5.1)$$

On generalizing the above sugeno example as follows:

Let the rules of the Sugeno FIS be of the form
R^i : IF (antecedent), THEN
$y^i = c_0^i + c_1^i x_1 + \ldots + c_m^i x_m$ for each rule i= 1..n……(5.2)

where y^1, … , y^n represent the consequents of each rule R^i in the rule base.

Suppose α^1 , …, α^n represent the firing levels of each rule. Then, Equation 5.2 takes the form –

$$Sugeno\ output = \frac{\sum_{i=1}^{n} \alpha^i y^i}{\sum_{i=1}^{n} \alpha^i} \quad \ldots\ldots\ldots\ldots(5.3)$$

Since our objective is to create a Sugeno FIS which produces results similar to that of equivalent Mamdani FIS,
Mamdani output = TSK output ⇔

$$\Leftrightarrow \quad Mamdani\ output = \frac{\sum_{i=1}^{n} \alpha^i y^i}{\sum_{i=1}^{n} \alpha^i} \quad \ldots\ldots\ldots\ldots\ldots\ldots(5.4)$$

On expanding Eqn 5.4,

$$\frac{\alpha^1 y^1 + \ldots + \alpha^n y^n}{\alpha^1 + \ldots + \alpha^n}$$

On further expansion,

$$\left[\frac{\alpha^1}{\alpha^1+\ldots+\alpha^n}\right]y^1+\ldots+\left[\frac{\alpha^n}{\alpha^1+\ldots+\alpha^n}\right]y^n \quad \ldots\ldots\ldots\ldots(5.5)$$

Let ω_i represent the normalized value for each α_i, i= 1,..,n
Then, Equation 5.5 reduces to

$$\omega_1 y_1+\ldots+\omega_n y_n \quad \ldots\ldots\ldots\ldots(5.6)$$

Thus, the output of the Sugeno FIS by substituting the consequent of each generalized rule y_i in Equation 5.6 gives

$$\begin{aligned}&(\omega_1)c_0^1+(\omega_1 x_1)c_1^1+\ldots+(\omega_1 x_m)c_m^1+\ldots+\\&(\omega_n)c_0^n+(\omega_n x_1)c_1^n+\ldots+(\omega_n x_m)c_m^n\end{aligned} \quad \ldots\ldots\ldots\ldots(5.7)$$

It is noted that there are m+1 constants corresponding to $c_{0\ldots m}^i$ for each of the ω_i for i=1,..,n. Thus, there are n(m+1) constants in total. Suppose we have a set of p values that can be used for the tuning process of the sugenomodel.Then, each of these n(m+1) constants represents an unknown vector that is to be determined by minimizing the difference in the results stored in the p values for the Mamdani FIS and Sugeno FIS.

This can be represented as an optimization problem as follows:

$$\begin{pmatrix} a_{11} & \cdots & a_{1,n(m+1)} \\ \vdots & \ddots & \vdots \\ a_{p1} & \cdots & a_{p,n(m+1)} \end{pmatrix}\begin{pmatrix} c_0^1 \\ . \\ c_m^n \end{pmatrix}=\begin{pmatrix} b_1 \\ . \\ b_p \end{pmatrix}$$

Or:

$$AX=\underline{b}$$

where,
A is the matrix whose row elements are ω_1, $\omega_1 x_1$, .. $\omega_1 x_m$ .Here, ω_i represents the normalized value for each α_i. $\underline{b}$ is a vector containing the p training values obtained from Mamdani FIS.X is the vector of unknown parameters.

Thus, the optimization problem now reduces to the following scenario. When p>n(m+1), the problem is said to be an Unconstrained non-negative linear least square problem. This problem, in turn can be solved by many heuristic algorithms such as genetic algorithm or Adaptive neuro-fuzzy inference method.

5.2 Non-negative Linear Least Square Algorithm(NNLS)

(Jassbi et. al., 2007) used the genetic algorithm and ANFIS method to solve the optimization problem. We propose to explore a new method to solve the unconstrained linear least square problem.

(Lawson & Hanson, 1974) proposed the non-negative linear least square algorithm.

The following are the steps to be followed:

1. Set all elements of **X** to zero, and set all indices into set *Z*, set *P* to empty set.
2. Compute the gradient vector **w** from current value of **x**: $\mathbf{w} = \mathbf{A}^T(\underline{\mathbf{b}} - \mathbf{Ax})$
3. If *Z* is empty or if all elements of **w** with indices in *Z* have values ≤ 0, terminate the algorithm.
4. Find the maximum positive element of **w.**Move its index from *Z* to *P.*
5. Create $\mathbf{A_p}$ from A where the columns corresponding to indices in *Z* are replaced with columns of zeros. Solve the unconstrained linear least squares problem $\mathbf{A_p z}=\mathbf{b}$. This will only determine the components of **z** corresponding to indices in *P*. Set the remaining elements of **z** to 0.
6. If all elements of **z** with indices in *P* are greater than 0, this is an acceptable new trial solution. Set **x** = **z** and go to step 2.
7. If all of the elements in **z** are not greater than 0, accept a fraction of **z** as the new trial solution. Find an index *q* Є P such that $x_q/(x_q - z_q)$ is the minimum for negative elements in **z**.
8. Set $\alpha= x_q/(x_q - z_q)$
9. Set $\mathbf{x} = \mathbf{x} + \alpha(\mathbf{z} - \mathbf{x})$
10. Move from *P* to *Z* all indices for which the corresponding element of **x** is zero. Go to step 5.

The NNLS algorithm is best suited for the given optimization problem. The NNLS algorithm is better than the least mean square method, genetic algorithm and ANFIS method used in [7]. The NNLS algorithm is very simple to implement. The NNLS algorithm is less complex than genetic and ANFIS algorithms and hence, consumes lesser computation time and resources.

5.3 Decomposition of MIMO into MISO systems

MIMO systems have multiple input and multiple outputs. One of the problems with TSK FIS is that it could handle only MISO systems.Hence, a procedure to convert TSK FIS from MIMO form to MISO is required.

A typical MIMO system is of the form –

" If x_1 is $A_{1,1}$ and x_2 is $A_{1,2}$ and … x_n is $A_{1,n}$, then $z_1=f_{1,1}(x_1,x_2,\ldots,x_n)$ and $z_2=f_{1,2}(x_1,x_2,\ldots,x_n)$ and … and $z_m=f_{1,m}(x_1,x_2,\ldots,x_n)$

and

.

.

and

If x_1 is $A_{p,1}$ and x_2 is $A_{p,2}$ and … x_n is $A_{p,n}$, then $z_1=f_{p,1}(x_1,x_2,\ldots,x_n)$ and $z_2=f_{p,2}(x_1,x_2,\ldots,x_n)$ and … and $z_m=f_{p,m}(x_1,x_2,\ldots,x_n)$"

where,

n is the number of inputs, m is the number of outputs and p are the number of fuzzy rules in the system.

Let us consider the first rule. This can be written as–

(x_1 is $A_{1,1}$ and x_2 is $A_{1,2}$ and … x_n is $A_{1,n}$) => ($z_1=f_{1,1}(x_1,x_2,\ldots,x_n)$ and $z_2=f_{1,2}(x_1,x_2,\ldots,x_n)$ and … and $z_m=f_{1,m}(x_1,x_2,\ldots,x_n)$) (5.8)

Without loss of generality, this is reduced to –

$(p_1 \text{ and } p_2 \text{ and } \dots \text{ and } p_n) \Rightarrow (q_1 \text{ and } q_2 \text{ and } \dots \text{ and } q_m)$ (5.9)

which is of the form a=>b imples ~a or b
$\sim(p_1 \text{ and } p_2 \text{ and } \dots \text{ and } p_n) \text{ or } (q_1 \text{ and } q_2 \text{ and } \dots \text{ and } q_m)$ (5.10)

$[\sim(p_1 \text{ and } p_2 \text{ and } \dots \text{ and } p_n) \text{ or } q_1] \text{ and } \dots \text{ and } [\sim(p_1 \text{ and } p_2 \text{ and } \dots \text{ and } p_n) \text{ or } q_m]$ (5.11)

$[(p_1 \text{ and } p_2 \text{ and } \dots \text{ and } p_n) \Rightarrow q_1] \text{ and } \dots \text{ and } [(p_1 \text{ and } p_2 \text{ and } \dots \text{ and } p_n) \Rightarrow q_m]$ (5.12)

On expansion of Eqn 5.12,
"If x_1 is $A_{1,1}$ and x_2 is $A_{1,2}$ and … x_n is $A_{1,n}$, then $z_1=f_{1,1}(x_1,x_2,\dots,x_n)$

and

.

.

and

If x_1 is $A_{1,1}$ and x_2 is $A_{1,2}$ and … x_n is $A_{1,n}$, then $z_m=f_{1,1}(x_1,x_2,\dots,x_n)$" (5.13)

Thus, the MIMO system can be decomposed into m MISO systems. This makes TSK FIS more attractive than Mamdani by eliminating another major drawback of the TSK FIS of not being operable with MIMO systems.

6. Experimentation

The following is the example[10] that would be used to be experimented with:

Consider a Mamdani Fuzzy Model for inferencing with the following rules:
i. If x is Small and y is Small then z is positive large
ii. If x is Small and y is Large then z is negative small
iii. If x is Large and y is Small then z is positive small
iv. If x is Large and y is Large then z is negative large
The antecedent fuzzy set memberships are defined as Small X = sig(x; -4, 1); Large X = sig(x; 3, 2); Small Y = sig(y; -4, 2) and Large Y = sig(y; 4, 1).

Fig 6.1 plots the membership functions of inputs X and Y using MATLAB® Fuzzy logic toolbox. Fig 6.2 shows the overall input output surface with max-min decomposition and centroid defuzzification.

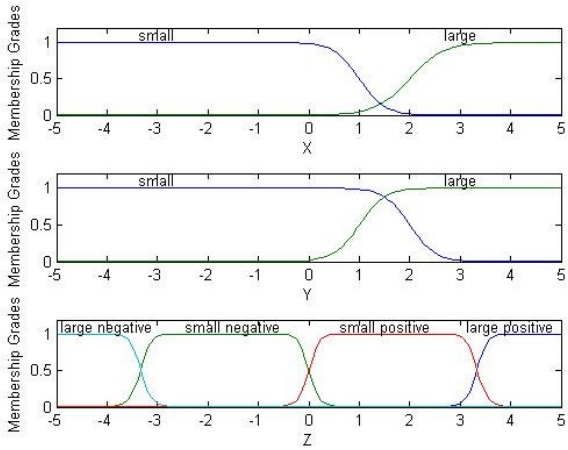

Fig 6.1 Rules of the Mamdani System

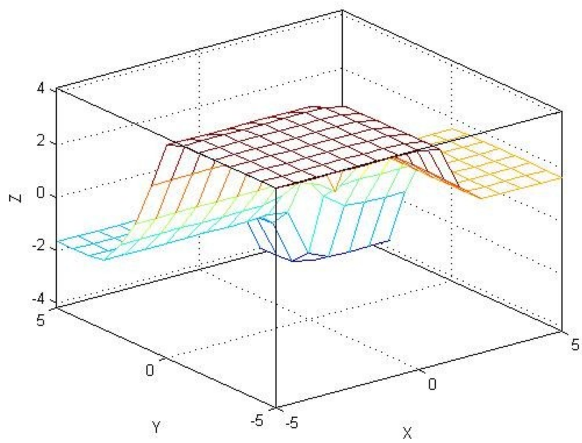

Fig 6.2 Surface of the Mamdani System

The values of c_j^i for i=1,…,n , j=0,…,n are to be found out for the above problem.

Acknowledgment

The author wishes to thank Prof. Shamik Sural, School Of Information Technology, Indian Institute Of Technology Kharagpur for providing a foundational basic and motivation to write this paper. This work was carried out as part of the curriculum for the course "Soft Computing Applications".

References

[1] J. Jassbi, P. Serra, R.A. Ribeiro, A. Donati, "Comparison of Mamdani and Sugeno Fuzzy Inference Systems for a Space Fault Detection Application", Proceeding of the 2006 World Automation Congress (WAG 2006)

[2] T.J. Meitzler, E. Sohn, "A Comparison of Mamdani and Sugeno methods for Modeling Visual Perception Lab Data", Annual Meeting of the North American Fuzzy Information Processing Society (NAFIPS 2005)

[3] A.Hamam, N.D. Georganas, "A Comparison of Mamdani and Sugeno Fuzzy Inference Systems for Evaluating the Quality of Experience of Hapto-Audio-Visual Applications", Proceeding of the 2008 IEEE International Workshop on Haptic Audio Visual Environments and their Applications (HAVE 2008)

[4] J. Jassbi, S.H. Alavi, P. Serra, R.A. Ribeiro, "Transformation of a Mamdani FIS to First Order Sugeno FIS", In: IEEE International conference on Fuzzy systems (FUZZIEEE07), 25-28, United Kingdom.

[5] E. H. Mamdani and S. Assilian, "An experiment in linguistic synthesis with a fuzzy logic controller", International Journal of Man-Machine Studies, Vol. 7, No.1, 1975, pp. 1-13.

[6] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control", IEEE Trans, on Systems, Man and Cybernetics, 15, 1985, pp. 116-132.

[7] J.-S. R., Jang, "ANFIS: Adaptive-Network-based Fuzzy Inference Systems", IEEE Transactions on Systems, Man, and Cybernetics, Vol.23, No. 3, pp. 665-685, May 1993.

[8] J.S. Jang, C.T. Sun and E. Mizutani, Neuro-Fuzzy and Soft computing, Prentice-Hall, pp. 73 – 91,1997.

[9] C. L. Lawson and R. J. Hanson. Solving Least Squares Problems. Prentice–Hall, 1974

[10] Question 2a, Mid Semester Question Paper, Soft Computing Applications, School of Information Technology, IIT Kharagpur,2009

Yogesh Piolet T is a student pursuing his postgraduate degree in Information Technology at the School of Information Technology in Indian Institute of Technology Kharagpur. The above work is presented as part of the curriculum for the course "Soft Computing Applications". He bears a registration number of 08IT6018.